Dr Margaret Lowenfeld Trust

Poleidoblocs 1964

By Dr Margaret Lowenfeld

Reprinted from "Mathematics Teaching" the Bulletin of the Association of Teachers of Mathematics No. 29, Winter, 1964

In setting out to teach mathematics to .children or to design a mathematics learning ' programme for a school system, the situation confronting the teacher, director of education or mathematics specialist differs profoundly from that lacing those, responsible for the teaching of such subjects as, for example, literature or history.

For centuries education has been carried on in these fields and the development in children, from primary school to university, of understanding and ability in these subjects has had time to ripen and diversity, so that a teacher starting at any point within the range of the system can to some extent see his own section in relation both to the pupils" past and future and follow the development of his subject from primary school to university.

Not so with the present-day teacher of mathematics. New types of thought are actively at work in mathematics as the situation of children is changing, the demands made on them by life are altering our knowledge of the processes of learning is growing and our concept of the ends we aim to achieve in mathematical learning is the subject of the liveliest controversy. How the parts are to fit into the whole we do not yet know. Nor are we sure of the dynamics that bring about in the child that development of enthusiasm for, and understanding of, mathematics that we aim to develop.

When, therefore, we set out to teach mathematics to children we confront a complicated problem.

We cannot any longer rely on the customary and the accepted, but if we are to be successful and to arouse in children an enthusiasm for mathematics there is a good deal we, need to rethink.

We have two aspects to consider: the nature of children and the nature and content of what we want to teach.

In 1959 an O.E.E.C. Conference at Royaumont stated the problem in the following terms:

'In the last 200 years—even in the last 100 I might safely say—more new mathematics has been discovered than in all the previous history of mankind. Yet until the most recent times only a small amount of this new mathematics has had any pronounced influence upon teaching below the post- graduate university level.' (*Par*. 20.)

'In truth, we find ourselves faced with an extremely urgent pedagogical problem. It is all too evident that in our primary schools we are failing to develop at all efficiently, or at all adequately, the latent mathematical talents and interests of the average child. Even worse, we arrive in far too many cases at the negative result of turning the pupil permanently away from the study of mathematics'. (*Par*. 51.)

and finally:

'It seems to me then that the first move should be to initiate the study of geometry on an intuitive basis in the very early primary years.' (*Par*. 56.)

To these Views we need to add the fact that a child's failure in mathematics can adversely affect his whole response to school work. '

Poleidoblocs arose out of this type of rethinking. They came slowly into being, arising out of study with .my co-workers of what children, well endowed intellectually but unable to do school work, would do with a variety of media in an atmosphere of enquiry. Watching them find their way through the confusions that separated them from their Childhood's eagerness, an understanding of what was going on in their minds gradually took shape and I found myself forming a hypothesis concerning the meaning of what these children that we were watching actually did.

When it was found that this hypothesis seemed to fit into the problems we were studying, Poleidoblocs were devised to test out whether what children and adults did with them fitted with what it was imagined that they might do.

A field was then looked for in which these tools might be more generally tested as to their power to 'develop (children's) understanding and their creative talents', as the Royaumont conference has it.

Continuous work with children was undertaken and in 1961 the British Association Unit for Research into the Generation of Mathematical Insights was formed, with Z. P. Dienes and myself as Directors. Throughout 1961/1962 work was carried on by the Unit in Leicestershire village schools. In 1962 I had the additional resource of consultation with Mrs. E. M. Williams who worked with us for some time on the Unit.

During the past five years approximately 1300 Poleidoblocs boxes have been applied for and sent to schools, training colleges, child guidance centres and university departments. The demand continues. Discussion of Poleidoblocs G and A and a comparison with seven other forms of structural mathematics material by D. Williams of N.F.E.R. will be found in Educational Research Vol. III Nos. 2 and 3 *Teaching Arithmetic by Concrete Analogy*—I. *Miming Devices*, and Educational Research Vol. IV No. 3 II. *Structural Systems*.

Poleidoblocs are, on the one hand, apparatus for introducing primary school children to mathematical thinking and on the other, tools for use in mathematical exploration during various periods of their school life. They are cut in shapes like those a child sees all around him and with built-in mathematical relations. The purpose of Poleidoblocs in the infants school is to put children into something like the situation of primitive mathematicians in that through handling and observing them. through using imagination and constructive ability in working with them. children have the opportunity to discover tor themselves mathematical properties both simple and more profound.

Poleidoblocs consist of two boxes of wooden blocks: one set coloured (G) and one set plain (A); see page 9. They represent an attempt to provide tools which will form a bridge between the thinking processes of the pre-school child and the new kinds of thinking he will meet in school. Poleidoblocs G presents sets in colour of the basic shapes, cubes, rectangular prisms of differing heights and lengths, triangular prisms, cylinders, cones and pyramids. The colours of the blocks act as hints to the children of correspondences between the blocks. The total contents of box G is 54 blocks. On the floor of the box is a diagram of the arrangement of the separate blocks. with their dimensions. The blocks are packed in a solid wooden box with slide-in lid. Poleidoblocs A consists of 140 rectangular and triangular prisms in plain wood varying in size from four blocks 2 in. X 5 in. X ½ in. to 24 ½ in. cubes. The blocks are so accurately cut that any block will fit on to any other block. At the bottom of this box, as of the G box. is a diagram of the arrangement of the pieces with their dimensions. The box containing Poleidoblocs A is the same size as that containing the G blocks, the G box is red in outside colour and the A box yellow.

Poleidoblocs G

The 4 red (2 in.) cubes are the first of a series of three, the other two being 4 cubes of 1% in. (green) and 4 of l in. (blue); three sets of four cuboids of 6 X 2 (yellow), 4 X 2 (green) and 2 x 2 (blue) all ½ in. thick form a series in area. Three sets of four cylinders 2 in. (red), 1% in. (green) and 1 in. (blue) in diameter fit on the faces of the relevant cubes, the colours giving the children a hint of their relations. The cylinders increase in diameter in arithmetic progression, the heights decrease in geometric progression. Although the 4 green cylinders have no simple mathematical correspondence as have the other two cylinders, they are of value in the study of series and in free construction.

Two additional red cubes are divided into quarters with two cuts running at right angles through the cube. in one case diagonally and in the other from the midpoint of each face, at right angles to one another.

The four blue 2 X 2 cuboids can be looked upon also as a division of the red 2 in. cube into four, by three cuts running parallel to the face of the cube. These divisions of the 2 in. cube thus give rise to two sets of four cuboids differing in shape, and four triangular prisms, all equal to one another in volume.

Finally are three cones (yellow), 2 in. in height with bases 1 in. in diameter, equivalent to the end faces of the blue cylinders and three pyramids (yellow), with base of one square in. fitting on top of the red pillars.

Experiment with the contents of this box will show the wide variety of interrelations that exist between the blocks. On the other hand none of these are immediately evident and to a child the blocks appear just as of great individual variety in colour and shape.

Poleidoblocs A

These consist of 4 of each of a set of rectangular prisms with cross section of 1 sq. in., 5, 4, 3, 2, 1 in. in length, with 24 small cubes of ½ in.; four sets of rectangular prisms ½ X 1 in. in cross section and 5, 4, 3, 2, 1 in. in length; four sets corresponding to the first set but ½ in. in cross section and 5, 4, 3, 2, 1 in. in length; four rectangular prisms 2 X 5 in. and ½ in thickness; four sets of right angled triangles 2 ½ in., 2 in., 1 ½ in., 1 in. on their short sides and ½ in. in thickness.

To the experienced eye it will immediately be apparent that a very large variety of fractional experiments and expressions are possible with this collection, as well as the simplest expressions of ordinal and cardinal number. Red squares of 1 in. supply tools for measuring and counting.

For more senior work a set of 6 regular and 6 irregular tetrahedra are supplied, relating to the 1 ½ in. cubes.

Turning from the tools to the children who will use them, let us see if we can get a picture of their situation as they come for the first time from home to school.

A child's relation to the world is an active one and at the same time one in which his senses are deeply involved. Small children tend to experience the outer world with several senses at the same time. A stone, for example, will have a 'feel', a shape, a smell and a colour. Children tend to identify themselves with what they handle. When this is a doll, a train, a Teddy bear, it is obvious; but when 'the other' is a construction they have made, the same is true.

There are in children many types of sensorial sensitivity, intellectual ability, eye-hand co-ordination, manual dexterity, enjoyment of finger manipulation, imagination, intuitive potentialities and previous interests. Each child has an urge within him to give outward form to his past and present experiences and to the pictures that take shape within him. To do so gives them reality for him and enables him to become master of them, instead of being dominated by them.

The way in which children react to the thoughts, drives, fears, hopes, experiences and ideas within them is in action: action, manipulation and experiment: copying, inventing, trying out, observing, reproducing: repeating and repeating until the problem or the play has lost its attractiveness. Children 'see' with their fingers. Indeed there is a certain correspondence between the way an archaeologist fingers and eyes a pottery shard he has just found and a child's examination of a pebble or a shell. The comparison of object with object, the ranging in shapes, the exploration of what they will do, what will fit on what: all come into a healthy child's play with objects around him. The modern artist seems to be driven by the same desire, to touch, turn, weigh, explore strange objects that he finds in order to compose them into a, for him, meaningful whole.

By nature a child of 5 is eager, responsive and active, but his immediate interests and responses are fluid, working simultaneously on several levels. To hold attention on one quality, for example length, is possible only for a short time; phantasy will enter in, amusement: or restlessness, or the wish to make something. A child has only a limited ability to focus on a given task: to eliminate disturbing impulses, or to select and make use of those faculties that will further the response he is about to make.

Children behaving spontaneously learn from adults and from each other, and do so by identification and by reconstruction; by seeing if they have understood something through an attempt to reproduce it, or by altering it to fit their wish for a world as they would like it to be.

Children react to their situation vis-à-vis adults in different ways. Respect for these differences enables children to reach their own individual development, and in so doing to enrich and enliven the total experience of the class.

Although the whole future of the child may depend on the teacher's recognition of these differences, this recognition may be made very difficult for her by the rigidities of classroom structure, the size of classes and the demands of the syllabus to be covered.

A procedure, therefore, designed to give equal opportunity to all children must take account of all this and provide adequately for the slow and the quick child, the bright and the ungifted, the restless and the attentive, to find their own individual ways to mathematical understanding.

We know very little about the way children think in their pre-school years. Perhaps our ignorance in this respect is due in part to the fact that our techniques of investigation are inappropriate for use with small children. The contemporary psychologist's approach to experimentation does not go very far towards helping us to sort out the complexities of children's cognitive development.

One of the most fruitful approaches to the study of the child has been that of Piaget. By placing the child in carefully contrived situations and observing his responses, Piaget has been able to arrive at certain conclusions about the child's development which do justice to some of its complexities. Although designed to accommodate a greater variety of responses from the child than is permitted by standard experimental techniques, the Piagetian procedure still circumscribes quite closely the child's behaviour, permitting the operation of only certain aspects of the child's multidimensional thinking.

If the problem be approached from another direction by the use of material with built-in mathematical relations, children can be allowed to make spontaneous use of the full spectrum of their creative and thinking abilities, because the structure of the material ensures that what is done with it is mathematically relevant.

In watching children at work with Poleidoblocs in free construction it becomes possible to see children's personalities, in action as it were. In order to help teachers to know what to look for and to keep a record, Record Forms were designed by the Unit.

In the summer of 1961, in New York, cinematograph studies were made of four 13 year old school boys constructing with Poleidoblocs. A one minute introduction of the material to the boys was made by me, the boys had eight minutes to work with the blocks, and a further minute occupied in discussion with me of what they had made. The boys worked individually, first with the blocks of Poleidoblocs G and then with those of Poleidoblocs A and finally as a group, with a combination of the two. Although the execution of this programme had a great many faults, yet the results suggest that valuable information about processes of thinking in children could be gained by cinematograph recording of children at work with this material.

To return to the question of difference in modes of response, which is central to our enquiry, mathematics at the primary school level is concerned with the child's assimilation of a network of simple mathematical relations which interact with one another. The adult approach to this situation is to sort these out into classes of relations of the same kind and to teach them separately, as for instance in the division into number, area, volume and so on. A child, however, acting spontaneously, reacts to the material before him in a global manner, his attention shifting about from one aspect to another and the child himself expressing in his work several aspects of himself simultaneously. His home life has, in a way, been giving him mathematical material. He counts his brothers and sisters and the sweets his mother gives him; he knows 'larger' or 'smaller' pieces of cake, he is aware to some extent of what 'will stay put' on top of what. But these pieces of knowledge arise from his affective relations with his surroundings and what we want to do for him is to enable him to ‘see’ and manipulate such notions in themselves.

In setting children to work, with Poleidoblocs, our aim is to create circumstances in which the children will be free to make their own acquaintance with these mathematically related blocks and through using and handling them, to find out these relations for themselves, at their own pace, and in their own way, using with this ‘finding out’ their creativity and manipulative ability, to the end that they experience satisfaction in the doing and pleasure in the learning that doing has brought them.

In this way a child becomes free from an external ‘right’ or ‘wrong’ and develops his ability to see whether what he has done is or is not the ‘right’ answer to a problem he has set himself or which has arisen out of his previous work.

Introduction of the Blocks to the Children

This is an important moment. It is essential that the children grasp from the beginning that Poleidoblocs are not just at attractive form of building bricks, but that they are, instead, something ‘special’ not seen outside the school and containing things that are exciting to find out.

In the *Manual for Teachers in Infant Schools* a full description is given of the mode of presentation of the blocks to the children which we have found achieves this end.

From this point on, the suggestion made at your last Conference that there should be it each classroom a ‘mathematics table’ is a valuable one. Associated with Poleidoblocs, this table should come to stand for the children as the place where they find out and carry out things for themselves, where they meet the teacher as the sharer and elucidator of their work, interested in their efforts and whose help they find leads them constantly to new discoveries, both of their own abilities and of the nature of objects they are working with.

The whole class having been introduced to the blocks and the blocks themselves put in a heap on the ‘mathematics table’, the children whose turn it is to work that day with Poleidoblocs, are asked ‘to make some- thing with the blocks and say when they are finished’. The children should then be left to get on by themselves. All work with Poleidoblocs at whatever stage in mathematical education they are being used, should start in this way.

Free Construction

INFANTS

The purpose of this stage is to give each child an opportunity to examine and experiment with the blocks of each box in turn; to become familiar with their general shape and colour and the ways in which they relate to and combine with each other, and then to use his individual powers in responding spontaneously to the opportunities they present.

As a child experiments with Poleidoblocs he finds that the blocks, as it were ‘talk back“ to him. G blocks piled in a heap just look attractive, but when several of them are pulled out and put in conjunction with one another in front of him they begin to ‘look’ like things and to suggest ideas. This is the point where it is possible to observe what the child has ‘brought with him’, as it were, to school. Colour photographs of children’s constructions of an entry class in Leicester- shire show an astonishing range of quality and type of construction in children of equivalent age and similarity of background. Some children seem to have an intuitive apprehension of symmetry and equivalences, some seem unable to achieve anything at all, failing for example to achieve equal lengths of opposite walls in a court-yard they are constructing. The essential point in assessing mathematically what a child has done, is to be certain of what the child’s View is of what he has attempted or achieved. With some children this is easy, they will talk continuously about what they are doing. With others considerable skill has to be used and the avoidance of a direct question.

The children’s comments upon what they are doing, their choice of type of construction, their degree of ability to settle down to a definite aim and to carry out what they aim to do, tells the teacher who understands the blocks how far they have come in readiness for learning, where their interest lies and to what extent they are able to mobilise that interest.

In experimental work with Poleidoblocs in schools, many types of groupings of children for free construction have been tried; children working alone, in pairs, in small groups, boys and girls together or boys separately from girls. When the children are combined in a group there arises a tendency for one child to dominate and others to copy. In other instances children of vivid personality may stimulate and inspire others or at times the whole group may combine in a corporate undertaking. The wise procedure is to vary the arrangements, children working at one time in one arrangement and at other times in another, individual work alternating with work in a group.

An interesting aspect of children's free constructions in the infants' school and relevant to our main theme, are constructions representing scenes in several dimensions, as 'a chimney smoking on top of a tower', 'Daddy's tool box', 'a hiding place', 'a tunnel', 'a tower', 'a station', 'an under- ground railway with a tent underneath and several small rockets'.

A second point of interest is implication of movement :—

> **Susan**, 8 years 6 months (with A blocks): 'water coming from another valley has to come through a directed channel. Gates to stop the water if it comes too quickly and at the end two gates open and let the water out'.
>
> **Kelvin**, 7 years 11 months (with G blocks): 'water and a bridge over it, speed boat going under three archways, two of which marked with buoys and couldn't be used'.

FREE CONSTRUCTION IN JUNIOR SCHOOLS

In Junior classes it is suggested that the children be presented with Poleidoblocs G to begin with, then Poleidoblocs A, then a combination of the two, three constructions being asked for with each. As Poleidoblocs A contain so many more blocks than Poleidoblocs G some discrimination needs to be used by the teacher as to the way these are to be used in relation to the particular children forming the class at any particular moment. *Only experience and an intimate knowledge by the teacher as the Poleidoblocs blocks* can be a safe guide.

Much can be discovered concerning the child's way of thinking by comparison between these three types of constructions. Some children will repeat the general form of their constructions whatever material they use. Others find one type of block easier to manage than another and will succeed with one to fail with the other. Some will embody the same idea in each type of material but carry it out in a different manner. All three kinds of exercise will give children practice and experience in perceiving the qualities of the individual shapes, used in different ways and seen from different angles.

Exercises in free construction are useful right through a school, in training colleges and with teachers. As adults we are so unused to seeing and thinking spatially, that the restricted nature of the collection of Poleidoblocs blocks forces the mind to observe and analyse. Particularly is this true with the two types of workers described by Dienes, the analytic and the constructive. Work with Poleidoblocs in a group of education students in training can enable students to recognise their own type of approach and to differentiate it from the opposite; also to gain experience of another way thinking than their own, which will give them insight later into the same variations in their children's work.

If teacher and child move together in discovery, then mathematics begins to come alive for both of them. So much of mathematics for adults has come to be taken for granted that in watching the child's gradual discovery of concepts long known, the concepts themselves come to have a new face.

It is taken for granted in this paper that other types also of mathematical teaching will be going on in the school, Poleidoblocs combine particularly well with the Dienes material. The children tend spontaneously to recognise the interaction of the blocks with other material. For example:

Anne and Lesley noticed how many very small units make rods ‘like our number apparatus for tens and units’.

The occurrence of spontaneous pieces of insight of this kind in children constructing with Poleidoblocs is an indication that they are ready for more direct mathematical work. ‘Before Anne started to build she sorted out different shapes and colours and put them into separate piles’.

‘Richard (5) constructed two bridges identical in shape and differing in size and said “the car goes through the big bridge and then the little one to get into the garage”.’

‘Geoffrey (5 ¼) said “I’m going to build a tower, I’m using all the square blocks up except the flat ones”: 4 of these (triangular prisms) make one whole square’.

Before moving on to consider direct mathematical teaching an apparent contradiction in what has been said needs to be discussed.

In the earlier part of this paper I have written about the child’s multidimensional or global mode of spontaneous thinking and reacting to objects and experiences. Piaget’s work stresses the child’s tendency to note only one aspect of an object or situation at a time, ignoring all other qualities. The existence of these two modes of reaction to the outside world are crucial and, I believe, are the key to many of our difficulties in teaching children mathematics. Careful observation of children in different situations will discover both reactions. The point of determination as to which will be present appears to lie in the nature of the situation. Part of the results of our experimental work in the treatment of children has been the discovery that when an experience impinges upon a child in such a manner as to hold his whole attention, it will he one aspect of that experience that registers in the child‘s mind and comes to stand for it in later combinations of that with other experiences. This is the position of the Child in Piaget‘s experimental situations. A problem is put before the child or a question asked, with care taken to exclude the distracting elements. When, on the other hand, objects are presented to a child without direction from the outside, in a neutral atmosphere, his earlier mode of reaction takes the field and he will respond with a multi-sensorial response.

Coming first to school, children learn with a global type of response to sensation. in their school years they substitute for this a kind of thinking where stage follows stage, structured according to accepted models. This is the adult mode of thinking and is the one that, for the most part, has hitherto been relied upon in mathematical teaching. It could be said, perhaps, that certain of the newer kinds of mathematical thought are more directly aligned to the child’s undeveloped conceptual apparatus. The simple notions of classification that underlie set theory are those which a Child will have needed to develop at a very early age in order to cope with his everyday experiences. Again the notions of position and correspondence that are basic to topology are also essential ingredients of a great deal of his perception and manipulation of the environment. It could be said perhaps that traditional arithmetic and Euclidean geometry are more remote from the natural schemata of a child’s response to his environment.

In deciding upon the manner in which to present mathematical experiences, indeed in deciding upon the mathematics to be taught, might it not be fruitful to aim at preserving the child’s earlier mode of response and interpretation, while at the same time introducing him to the desired cognitive disciplines? Work with Poleidoblocs in the junior school is an experiment in this direction.

Here symbols are not used, but the children are required to draw round the models they construct and to keep these drawings in their books. Later symbols will be given to the same drawings. and with care and thought, adjusting the work to the pupil’s abilities, arithmetical and algebraic symbols can both be utilised.

Direct Mathematical Teaching

For example let us take the first piece of work:

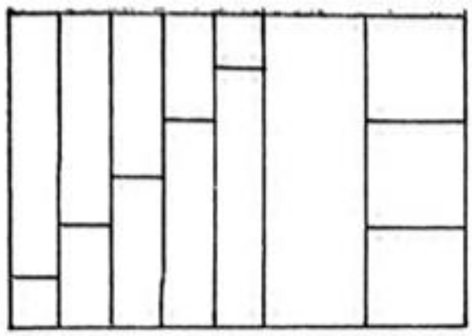
We start with the flat half-inch-thick blocks of Poleidoblocs G and A and consider the numbers 6 and 2. These are expressed in area form in the yellow slats of Poleidoblocs G. Numbers 6 and 2 have been chosen as a base because of their valuable relationship to the ideas and experiences of cubing and squaring, both of arithmetical and of geometrical quantities. 2 + 2 + 2 = 6 as in the long measure of the yellow slat but, as the child will later find, 2 X 2 X 2 becomes 2^3 as expressed in the red two-inch cube. Similarly 2, the length in inches of the shorter side of the yellow slat, appears also as the length of the side of the blue square. In fact, 2 X 2, as the children will discover, expresses the area of the blue square.

A further example of this area will appear when the four yellow slats are put together in such a way that the space enclosed by the four sides is also a 2 X 2 square area.

The teacher having explained that *it is the lengths of the edges of the yellow slats that they will be considering*, beginning with the long sides (i.e. focussing their attention upon this quality only of this block), shows them that three blue cuboids from the G box, if placed alongside the edge of the yellow slat, exactly fit its length.

The children are then invited to select as many as they like of the half-inch thick, one- inch wide blocks of Box A, to see in how many different ways they can arrange these to make up the length of the side of the yellow slat, using only two blocks at a time, and arranging them along the side opposite to the blue squares. The teacher demonstrates five plus one as an example of what can be done and suggests that there are five ways in which the desired end can be achieved.

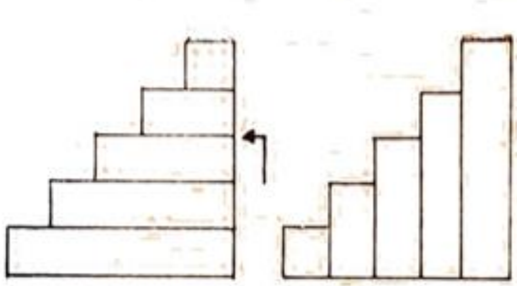
The children should be encourage-d to find their own way of arranging different combinations of the one-inch wide piece-s which fulfil the condition of making up together the length of the yellow slat. It is useful for them, at this point, to work in pairs. The goal of the exercise, however, is that the children should arrive at arranging the blocks in ascending or descending order so that the following picture is obtained.

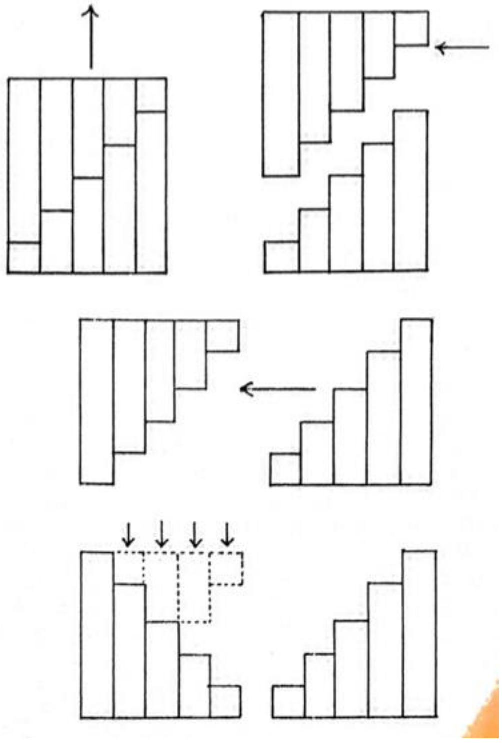


The concept of pattern is fundamental to mathematics. This is a moment when the children can discover for themselves that the A blocks which they have arranged to one side of the yellow slat make a pattern in their horizontal divisions. The teacher should demonstrate that, if the lower five blocks be separated slightly from the top group of five, a 'staircase' appears.

If the work in the infants' classes has been successful, this pattern will 'chime' with work they have already done when considering ordinal number series, or, counting and measuring.

At each stage the teacher should not proceed until she is sure that the children have grasped the point in hand. From here the work is carried on in arrangement and re-arrangement of the blocks of the 'staircase' pattern.





With each change, the pattern should be redrawn in their books. The children should be encouraged to practise the construction and re-arrangements of these patterns until they are fully familiar with them and can construct them at will.

A similar process is now carried out with the short side of the yellow slat.

Attention is now directed to squares. The children should be asked to compare the yellow slat and the blue square, and to find for themselves that. while the sides of the yellow slat are of different lengths, all four sides of the blue square are of the same length. They should then he asked to put one blue square upon another; then turn the upper one to see that each time an ‘upper’ corner meets a ‘lower’ one, the corners fit together perfectly. They should then be asked to draw a corner of the blue square in their books, and see if the blue square can be fitted against it with every one of its ‘real’ four corners. The teacher should then explain that *this type of shape is called a square.*

Work now moves to the study of square 8 and solids with square faces, and bit by fit the characteristics of a square are formulated: that there are four sides; that all of these are of the same length and equal to one another; each corner can be fitted to the same corner space. The teacher tests their grasp of this by putting on the table various arrangements of slats that make rectangles which are not squares and asking the children's opinion as to whether these are or are not squares and why. The children are then asked to make different sized squares with the blocks, and to experiment further with the number of 'squares' which are needed to cover other squares. They should be asked, also to make square spaces with the blocks of Poleidoblocs A.

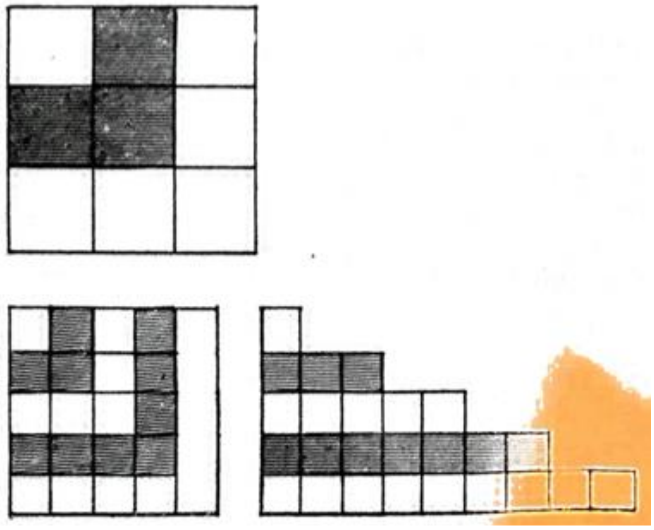
This concrete work is now turned into its number aspect, the teacher saying 'If one takes the edge of the blue block as 1, what will the edges of the other squares be?'

Play, either free or as competitive games of the cribbage type, is useful here, to test out how far all the children have understood the work they have been doing.

Series. The squares can now be arranged in series and the work done on series in infants classes revised. In constructing the series, the number values can be shifted and the l in. squares from box A brought in, with the explanation that it is the top surface of these that is being thought about. It can be shown that the number values given are not a property of the blocks, but an agreement between teacher and children.

The bright and the slower children here differentiate themselves. The brighter children can be given further work with the making of larger squares, the slower, repetitive work with the stage they have reached. Here the red squares function as measurers.

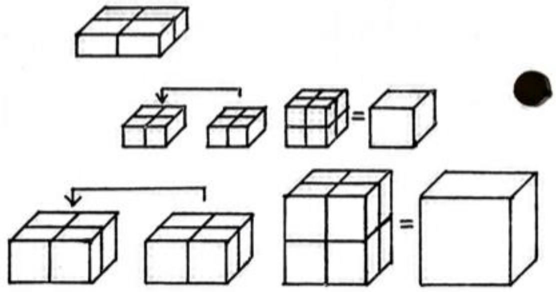
The series so far constructed are in two dimensions. Work is now carried to three dimensions by the putting of smaller squares, in series, upon larger ones.



Study of the spaces of each colour left uncovered, measurement of them with the square red pieces, and rearrangement, reveals a series of odd numbers. The qualities of these can then be examined. Having studied squares we turn to rectangles that are oblongs and study the detailed differentiation between these two kinds of rectangles. At this point combination with work with Dienes' equilateral triangles on the arithmetic concept of 'square' is particularly valuable.

From squares work moves to the study of cubes. In their infants classes the children have already worked with the composition and decomposition of the red 2 in. cube, and this work can be revised. The study of cubes . then begins. The four blue slats and the eight blue cubes and their equivalents in the A box cubes should be spilled out and the children invited to construct other cubes with these. From their results the characteristics of the cube are worked out in the same manner as has been done with squares.

To consolidate this work further a considerable period is spent in the study of equivalences between alternative shapes of blocks that have been used in the construction of cubes, for example four blue cubes/ two blue square cuboids or 2 blue cubes/one red rectangular prism. Finally a series of cubes is constructed.



An essential point upon which we badly need information is the relation between the children's work on the lines described above and their free construction. There is a fairly consistent opinion of teachers working in junior schools that, with time, free constructions show development, but at present we have no evidence to show how far this is genuine learning of mathematical relations and how far the effect of maturation and practice.

To return to the question of direct mathematical teaching in the infants schools. What is it, basically, that we want children in infants schools to obtain a grasp of P The answer seems to me to separate into two groups:

A. The nature and properties of shapes, in themselves and in interrelation, as for example.

    i. The properties of shapes: the basic discrete shapes as such and many varieties of these seen in various orientations, used for specific purposes in constructions and the discrimination between positions.

    ii. Relationships between shapes: fitting, matching, covering, filling the same space; the identity and equivalence of length, area, volume.

    iii. Equivalence of combinations of shapes.

    iv. Comparison of shapes: greater than, less than, the shape required to give completion; composition and decom- position of shapes.

The basic shapes of cubes, cuboids, cylinders, cones and pyramids are present in Poleidoblocs G and cuboids of many types in the A box. The faces of the pyramids are tri- angles, the faces of the cylinders circles of different sizes, and triangles of several sizes are present in the A box. If the children are working also with the Dienes material they will be familiar also with equilateral triangles. It is probable that the children in other aspects of their school work will be meeting two dimensional shapes in various ways that correspond with those met with in Poleidoblocs.

Space does not permit the discussion of methodology in direct teaching of the children about the relations between these shapes and their characteristics. In practice individual teachers will develop their own applications. One of the values of Poleido- blocs is that they enable teachers to test how far children have really understood the work they have been doing.

B. Concerning the four rules.

i. Ordination of similar shapes (Poleidoblocs A) and much play, back and forth with staircase phenomena, work with sequences.

ii. Practical addition and subtraction.

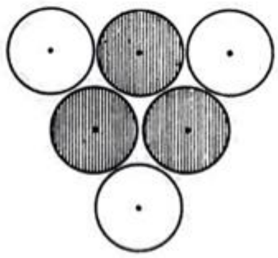
iii. Simple modes of practical division.

iv. The effect of iteration of identical shapes and the operation of multiplication.

Here an important principle enters. Since the four rules are the same whether applied to volume, area, length, or number, concrete material of the Poleidoblocs type makes it possible to work interchangeably in any of them. The concreteness of the material makes it possible also to use stories from the children's own experience to embody the steps in study of the four rules, taken, at this stage, in very simple form.

CONES, PYRAMIDS AND TETRAHEDRA

Apart from their value in free construction, where children of all ages make extensive use of them, and apart from their use as examples of basic shapes, the cones and pyramids lend themselves to extension of the children's experience of the less obvious aspects of mathematical relations. Six cones placed in the pattern illustrated below, for example, enable children to explore the properties of an equilateral triangle where the inner and the outer three give inversions of position, etc.



Six pyramids placed together side by side with points touching, produce the half of a dodecahedron with a radius of 2 in.

The tetrahedra do not come into use until the upper class of the Junior school when they extend the pupils' experience of the composition of complex shapes.

In this account it has only been possible to sketch in a very general and tentative way, an outline of the uses to which these materials can be put. Work with Poleidoblocs is still at the experimental stage and the possibilities latent in the material and the experience out of which they arose, only partially explored.

(*The Poleidobloc material is available through the Educational Supply Association, Harlow, Essex*).